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Space Charge Mismatch for the Transition Energy Crossing in the AGS and RHIC

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One of the problems encountered in crossing the transition energy in the AGS or RHIC is the mismatch caused by the space charge forces acting between particles of the same bunch. The mismatch is commonly measured by the Sörenssen parameter η_o, which is the ratio of the space charge forces to the focusing forces of the external rf cavities.

We estimate here the Sörenssen parameter η_o adding also the contribution of the "inductive wall" to the space charge, and for bunches with gaussian distribution. Let

$$Z_n/n = i \frac{Z_o g}{2\beta\gamma^2} - i\omega_o L = - i\omega L_{eff}$$

where the first term is the space charge proper and the second represents the "inductive wall" contribution. In Eq. (1)

Z_o = 377 ohm

β,γ = velocity and energy relativistic factors

g = 1 + 2 log (b/a)

b = pipe radius

a = beam radius

ω_o = 2π x revolution frequency

L = wall inductance

If the beam bunch is made of ions of charge state Q, the energy gain (loss) per turn is

$$\Delta E_{sc} = Qe \sum_n Z_n I_n e^{in(\theta-\omega_o t)}$$

$$= - Qei\omega_o L_{eff} \sum_n n I_n e^{in(\theta-\omega_o t)}$$

$$= - Qe\, \omega_o L_{eff} \frac{\delta}{\delta\theta} I\,(\theta-\omega_o t)$$

for a particle located at $\theta-\omega_o t$ within the beam current distribution $I(\theta-\omega_o t)$. Because of the periodicity with θ, the orbit angular coordinate, the bunch current has Fourier coefficients I_n. Let

N_B = number of particles per bunch

σ_e = rms bunch length

R = average radius of closed orbit

then we can take

$$I(\theta) = \frac{Q\,\beta c\ e\ N_B}{\sqrt{2\pi}\ \sigma_e}\ e^{-\frac{\theta^2}{2\theta_o^2}}$$

where $\theta_o = \sigma_e/R$ and

$$\Delta E_{sc} = \frac{Q^2 \beta c\ e^2\ N_B\ |Z_n/n|}{\sqrt{2\pi}\ \sigma_e\ \theta_o^2}\ \theta\ e^{-\theta^2/2\theta_o^2}$$

For small oscillations amplitude the equivalent contribution from the external rf cavities is

$$\Delta E_{rf} = QeVh\ (\cos\theta_s)\ \theta$$

In proximity of the bunch center ($\theta \ll \theta_o$), the Sörenssen parameter finally is

$$\eta_o = \frac{Q\beta ce^2 N_B |Z_n/n| R^2}{\sqrt{2\pi}\ \sigma_e^3\ ehV\ |\cos\theta_s|} \tag{1}$$

where

θ_s = rf phase angle

V = rf voltage

h = rf harmonic number

The original theory by Sörenssen assumed a uniform bunch distribution, thus Eq. (1) is expected to yield a larger value for η_o because of the assumed gaussian distribution and for the same overall beam dimensions.

AGS

We have calculated the Sörenssen parameter according to (1) and show the results in Figures 1 and 2. The parameters used are as follows

γ_T = 8.5

R = 128.5 m

V = 0.3 MV for protons , 0.24 MV for gold

θ_s = 36° 20°

b = 10 cm

h = 12

No γ_T-jump or other ways to speed up the crossing of the transition energy have been assumed.

The η_o parameter is displayed in Figures 1 and 2 versus the rms bunch area S_o for different number of particles per bunch N_B. As one can see from (1), η_o is linearly proportional to N_B and the coupling impedance $|Z/n|$. For each case two estimates were made: one which corresponds to pure space charge forces (S.C.) and no inductive wall (L=0), and the second for a total $|Z/n|$ = 8 ohm. According to E. Raka an inductive wall contribution of 20 ohm can be derived from beam observations in the AGS. When from this one subtracts the space charge contribution of ~ 12 ohm, the value of 8 ohm is then obtained.

The usual requirement to avoid exceedingly large mismatch at transition crossing is $\eta_o < 1$. This in turn can set a limit of the bunch area S_o for a given intensity N_B which can be derived by inspection of Figures 1 and 2 with the following results:

For protons

N_B	6 S_o for $\eta_o \sim 1$
1×10^{11}	0.4 eV.sec
2	0.6
4	1.0
1×10^{12}	1.8

For gold

N_B	6 S_o for $\eta_o \sim 1$
1.1×10^{9}	0.35 ev.sec/amu
2.2	0.5

RHIC

The results for RHIC are shown in Figure 3 for the case of gold with 2.2×10^9 ions per bunch. Other parameters are

$\gamma_T = 24.5$

$R = 610.2$ m

$V\sin\theta_s = 0.048$ MV/turn

$b = 3.6$ cm

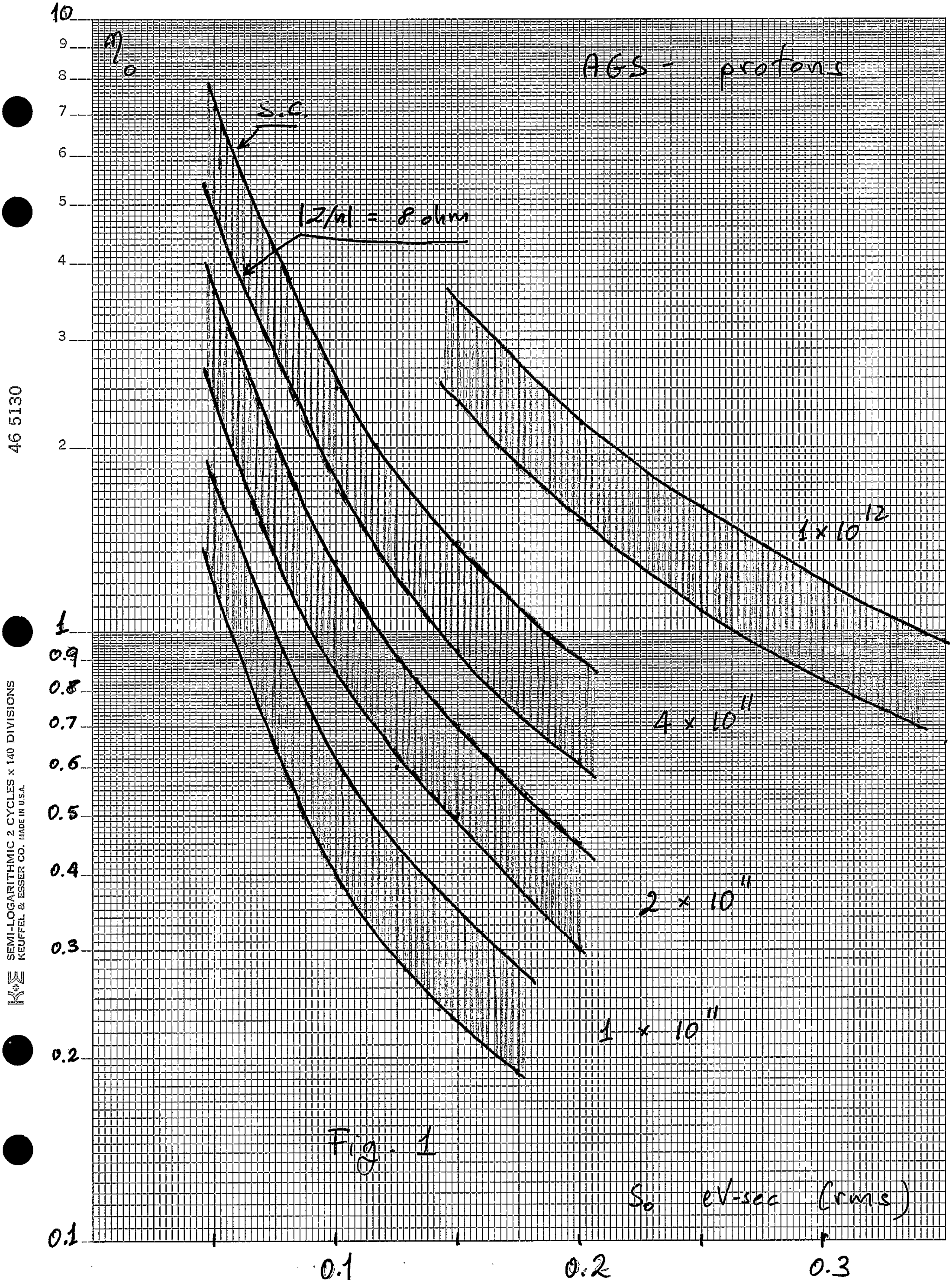
$h = 342$ (26 MHz)

Again, no γ_T-jump has been assumed.

The space charge contribution to the coupling impedance is 1.4 ohm. We made two estimates, one with only space charge terms and the other with a total $|Z/n| = 5$ ohm. As one can see, for a given bunch area and intensity, the η_o parameter is larger in RHIC when compared to the AGS case. In order $\eta_o < 1$ the bunch area 6 S_o should be 1-3 eV.sec/amu depending on the coupling impedance assumptions.

The situation would be considerably worst for the case where also proton bunches would be required to cross the transition energy, as it has been proposed recently for some rf schemes. The results for protons with 2×10^{11} particles per bunch are given in Figure 4. Depending on the value of the coupling impedance, 6 S_o = 2-5 eV.sec in order $\eta_o < 1$.

If the rate of transition energy crossing is increased with a suitable γ_T-jump scheme so that $\dot{\gamma}_T$=20 sec^{-1}, then the range of values of the bunch area (6 S_o) are reduced to 0.5-1 eV.sec/amu for the case of gold and 1-2.5 eV.sec for the case of protons.



η_0
AGS - protons
S.C.
$|Z/n| = 8$ ohm
1×10^{12}
4×10^{11}
2×10^{11}
1×10^{11}
S_0 eV-sec (rms)
10
9
8
7
6
5
4
3
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.1
0.2
0.3
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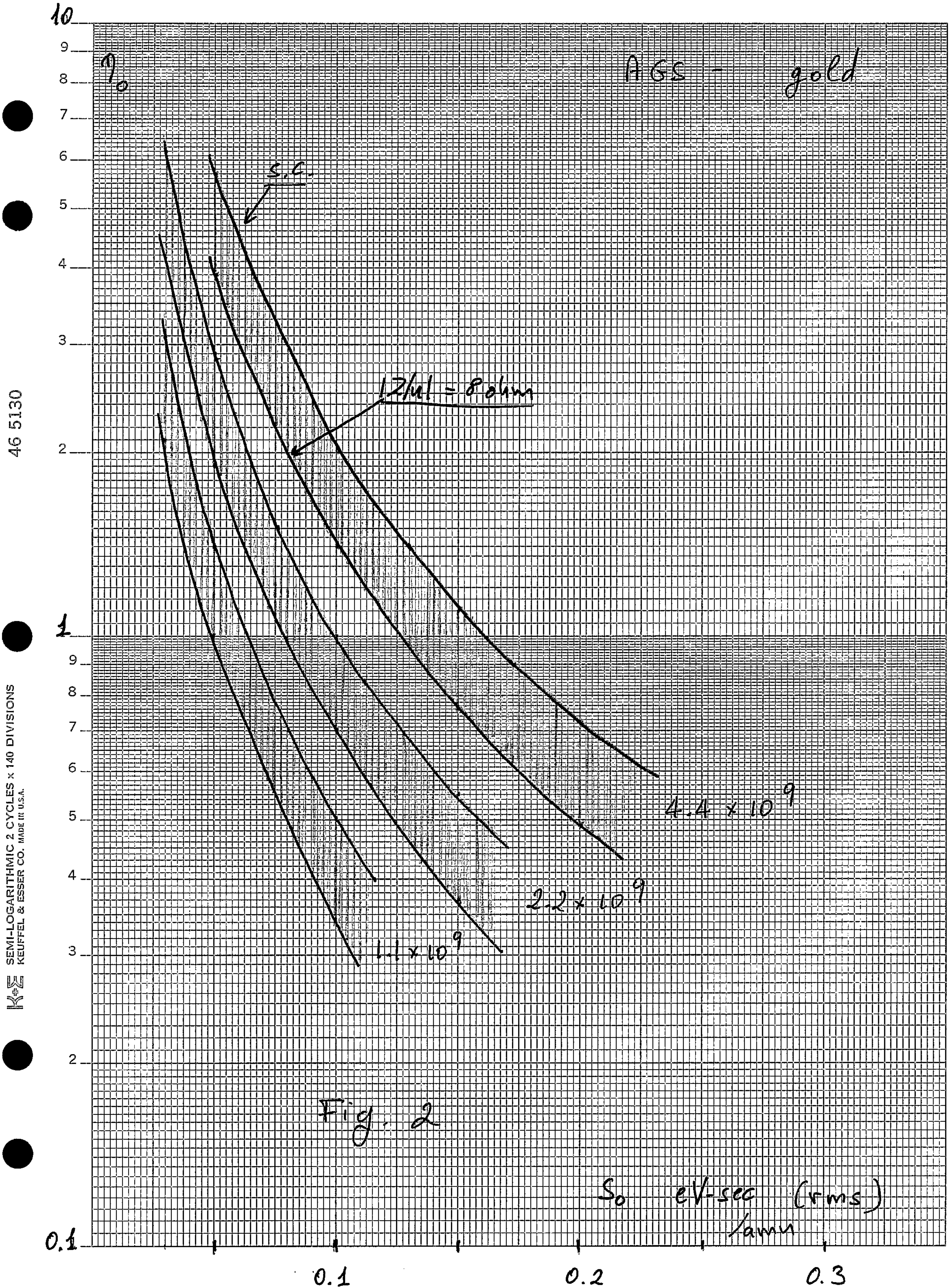
SEMI-LOGARITHMIC 2 CYCLES x 140 DIVISIONS
KEUFFEL & ESSER CO. MADE IN U.S.A.

Fig. 1



η_0
AGS - gold
s.c.
|Z/n| = 8 ohm
4.4 × 10⁹
2.2 × 10⁹
1.1 × 10⁹
S_0 eV-sec/amu (rms)
10
1
0.1
0.1
0.2
0.3

Fig. 2

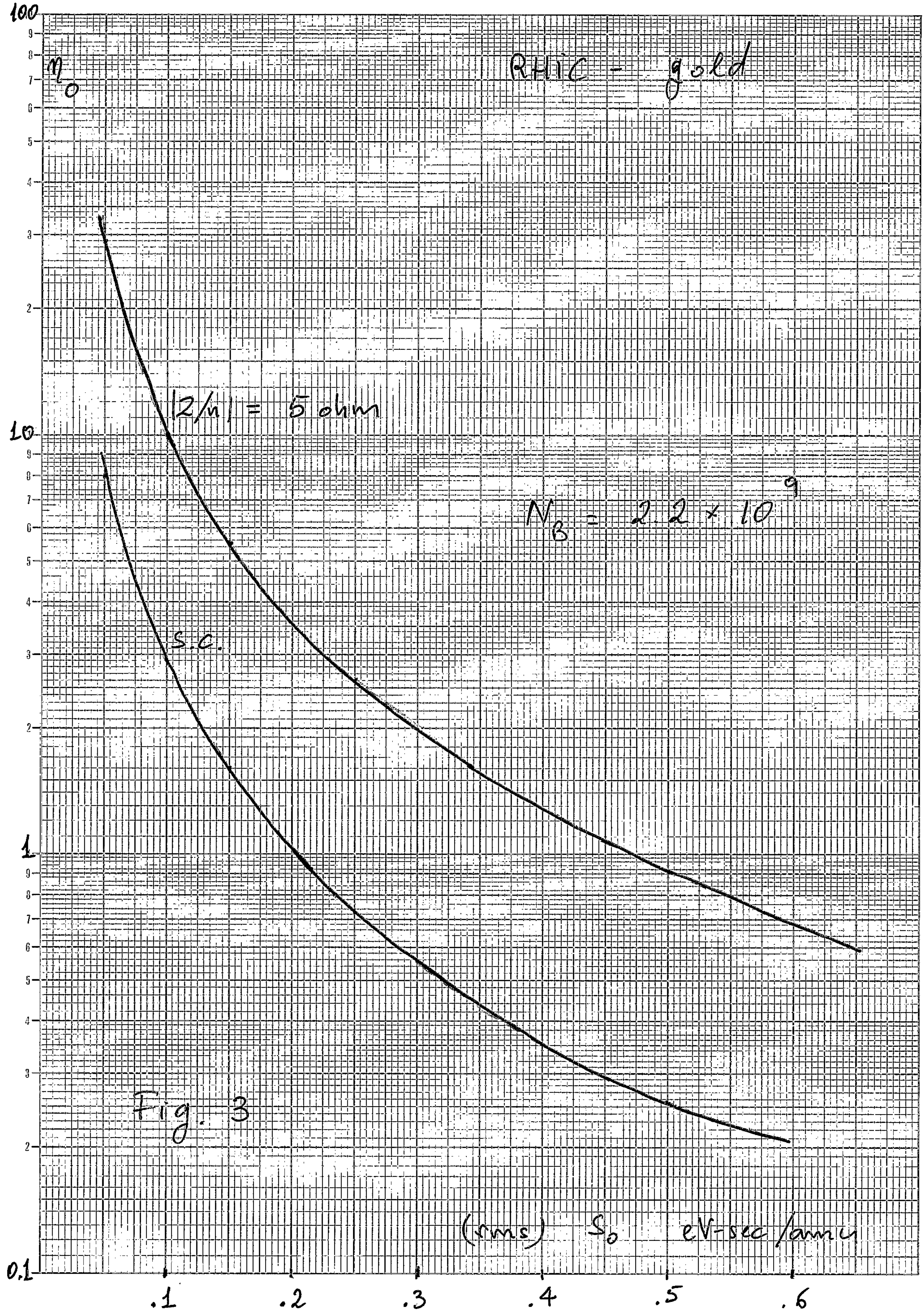

RHIC - gold
η_0
$|Z/n| = 5$ ohm
$N_B = 2.2 \times 10^9$
s.c.
(rms) S_0 eV-sec/amu
100
10
1
0.1
.1
.2
.3
.4
.5
.6

Fig. 3

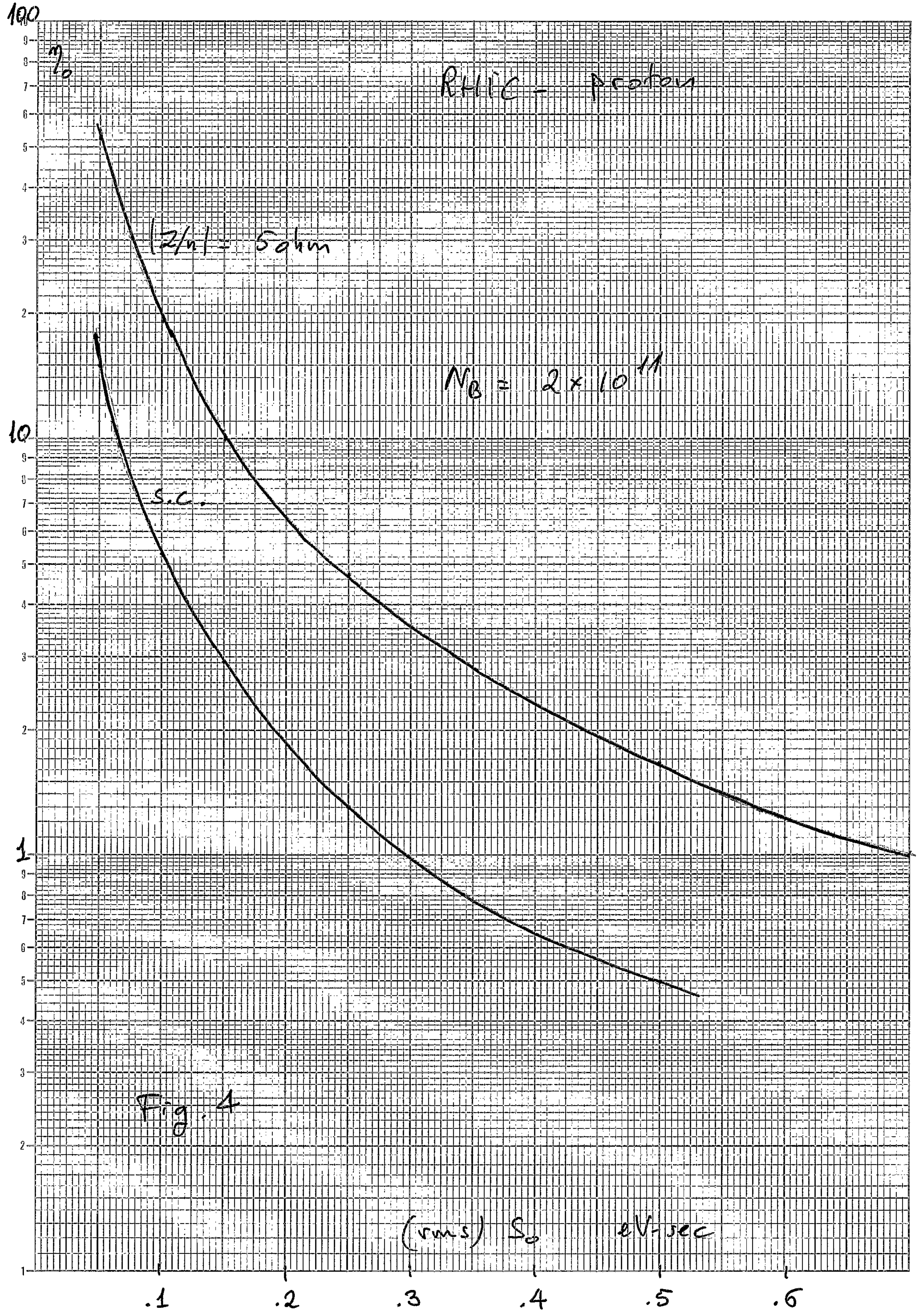


Fig. 4